D0-branes in black hole attractors

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ABSTRACT: Configurations of N probe D0-branes in a Calabi-Yau black hole are studied. A large degeneracy of near-horizon bound states are found which can be described as lowest Landau levels tiling the horizon of the black hole. These states preserve some of the enhanced supersymmetry of the near-horizon $AdS_2 \times S^2 \times CY_3$ attractor geometry, but not of the full asymptotically flat solution. Supersymmetric non-abelian configurations are constructed which, via the Myers effect, develop charges associated with higher-dimensional branes wrapping CY_3 cycles. An $SU(1,1|2)$ superconformal quantum mechanics describing D0-branes in the attractor geometry is explicitly constructed.

KEYWORDS: Black Holes in String Theory, D-branes.

Contents

1. Introduction

Despite a number of exciting chapters, the story of Calabi-Yau black holes in string theory [1] remains to be finished. There are several indications of this. One is that we have yet to understand the expected [2] superconformal quantum mechanics which is holographically dual to string theory in the near-horizon $AdS_2 \times S^2 \times CY_3$ attractor geometry. A second is the mysterious and recently discovered connection [3] relating the black hole partition function to the square of the topological string partition function, which in turn is related to the physics of crystal melting [4]. All of these point to a surprise ending for the story.

The chapter of interest for this paper involves the study of D-branes on type II Calabi-Yau black holes with RR charges. This is relevant for the proposal, pursued in a companion paper [5], that the dual superconformal quantum mechanics may be realized as D-brane

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[1]An excellent recent review can be found in [1].

quantum mechanics in the near-horizon attractor geometry. Branes also play a role in the connection of [3] and are related to the atoms of the crystals appearing in [4]. Some development of this chapter appeared recently in [6], where a rich variety of classical, single-brane configurations were found which preserve some supersymmetry of the near-horizon attractor region but not of the full black hole geometry.

In this paper we continue this chapter, expanding the classical analysis of [6] by studying D0-brane configurations, including quantum effects as well as non-abelian effects for multiple D0-branes. One interesting result is that, due to magnetic RR fields, the D0-branes probe a non-commutative deformation of the attractor geometry. This leads to a large degeneracy of near horizon D0 bound states, which can be described as lowest-Landau-level states which tile the black hole horizon.[2] This is suggestive of the old idea that the area-entropy law results from planckian degrees of freedom which tile the horizon with planck-sized cells.

This paper is organized as follows. Section 2 describes large N non-abelian collections of D0-branes which grow charges associated to branes wrapping higher dimensional Calabi-Yau cycles. Parts of the relevant mathematics - geometric quantization, deformation quantization and noncommutative geometry - are briefly reviewed in section 2.1. Section 2.2 embeds these mathematical structures in the problem of N D0-branes. Supersymmetry (or the equations of motion) of the D0 configuration gives an interesting constraint on the noncommutativity parameter $\theta \sim \omega^{-1}$, where ω is a closed two-form. It is found that, to leading order at large N, the Ricci-flat Kähler form J on CY_3 must be harmonic with respect to the metric associated to ω. This result is shown to agree with that obtained from a DBI analysis of the D6-brane generated by the D0-brane configuration. In 2.3 formulae are given for the induced higher-brane charges. In 2.4 an explicit example at finite N ($N = 5$) of a non-abelian D0 configuration on the quintic is given. In section 3 we turn to D0-branes in the $AdS_2 \times S^2 \times CY_3$ attractor geometry. In 3.1 we show that the D0 ground states (with respect to the global AdS_2 time) are lowest Landau levels which tile the black hole horizon. The ground state degeneracy is the D6 flux though the horizon, which we denote p_0. In 3.2 we construct the bosonic conformal quantum mechanics describing a D0-brane in $AdS_2 \times S^2 \times CY_3$. As a warm up in 3.3 we supersymmetrize the $AdS_2 \times S^2$ part. In 3.4 the full superconformal quantum mechanics for D0 branes in the attractor geometry is constructed. The chiral primary states are described in 3.5. In 3.6 they are summarized by an index. In 3.7 we consider the interesting case a D2 brane with N units of magnetic flux (i.e. an N D0 brane bound state) wrapping the horizon S^2. The corresponding superconformal mechanics, which plays a role in the companion paper [5], is constructed.

2. N D0-branes in $R^4 \times CY_3$

In this section we describe configurations of N D0 branes which, via the Myers effect [8] carry charges associated with higher dimensional branes wrapped around cycles of a Calabi-

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[2]Related phenomena were encountered in attempts to embed the quantum Hall effect into string theory [7].

Yau threefold, denoted CY_3. We will also analyze their supersymmetry properties in the asymptotic $R^4 \times CY_3$ region far from the black hole where background RR fluxes can be ignored. Near horizon brane configurations will be considered in the next section.

A collection of N D0-branes with the $N \times N$ matrix collective coordinates Φ^A, $A = 1, \cdots 9$ can source higher dimensional brane charges, through the Chern-Simons terms [8],[9]

$$
S = T_0 \int dt \mathrm{Tr} \left\{ C_t^{(1)} + i\frac{\lambda}{2}[\Phi^A, \Phi^B](C_{tAB}^{(3)} + C_t^{(1)} B_{AB}) \right. \tag{2.1}
$$
$$
\left. - \frac{\lambda^2}{8}[\Phi^A, \Phi^B][\Phi^C, \Phi^D] \left(C_{tABCD}^{(5)} + C_{t[AB}^{(3)} B_{CD]} + \frac{1}{2} C_t^{(1)} B_{[AB} B_{CD]} \right) + \cdots \right\},
$$

where $\lambda = 2\pi\alpha'$. In this expression $C_{\cdots}^{(p)}$ are the various p-form RR potentials. We will largely consider the case where the NS potential $B = 0$. We are interested in noncommutative D0 brane configurations in CY_3 of the form

$$
[\Phi^A, \Phi^B] \sim \theta^{AB}(\Phi), \tag{2.2}
$$

where $\theta^{-1} \sim \omega$ is a non-trivial two-form. In order to give a precise version of equation (2.2), it is convenient to employ the framework of geometric quantization, which we now review.

2.1 Brief review of geometric quantization and the star product

Berezin's geometric quantization [10, 11] of a Kähler space M describes the quantum mechanics of a particle whose phase space is M. In this paper we will be interested in the special case when $M = CY_3$ is a Calabi-Yau threefold. The quantization begins with a choice of holomorphic line bundle $\mathcal{L}$ over M, with metric e^{-K} where K is the Kähler potential associated with a chosen Kähler form ω. This is possible only if $[\frac{i}{2\pi}\omega] = c_1(\mathcal{L})$ is integral. ω will turn out to be roughly θ^{-1}. The space of holomorphic sections of $\mathcal{L}$, $\mathcal{H} = H^0(M, \mathcal{L})$ is then a Hilbert space with inner product defined by

$$
\langle s_1 | s_2 \rangle = \int_M \bar{s}_1(\bar{z}) s_2(z) e^{-K} d\mu \tag{2.3}
$$

where $d\mu = \frac{1}{3!}\omega^3$ is the volume form associated with the Kähler form ω. $\mathcal{H}$ is finite-dimensional with dimension given by the Riemann-Roch formula

$$
N \equiv \dim \mathcal{H} = \chi(\mathcal{O}_M(\mathcal{L})) = \int_M \frac{1}{6}\omega^3 + \frac{1}{12}\omega \wedge c_2(M) \tag{2.4}
$$

Let s_k be a basis of holomorphic sections that are orthonormal with respect to (2.3). Then, given an operator $\mathcal{O}$ acting on $\mathcal{H}$, one can get a function (covariant symbol) on M associated with $\mathcal{O}$ via,

$$
f_{\mathcal{O}}(z, \bar{z}) = \frac{\sum_{i,j=1}^N \bar{s}_i(\bar{z}) \mathcal{O}_{ij} s_j(z)}{\sum_{i=1}^N |s_i(z)|^2}, \tag{2.5}
$$

where

$$
\mathcal{O}_{ij} = \langle s_i | \mathcal{O} | s_j \rangle \tag{2.6}
$$

is $\mathcal{O}$ represented as an $N \times N$ matrix. This gives only N^2 independent functions, so not every function on M is the symbol of an operator on $\mathcal{H}$. The space of smooth functions is formally regained from the space of covariant symbols in the limit $N \to \infty$.

Instead of working with operators, one can formulate this as noncommutative geometry on M, in which the ordinary algebra of functions is replaced by the noncommutative $*$ algebra which reflects the operator algebra on $\mathcal{H}$. This is defined for all functions in the sense of deformation quantization [12, 13], i.e. in a formal power series expansion in $\frac{1}{N}$, or equivalently the noncommutativity parameter θ or ω^{-1}. Hence the noncommutative geometry picture is good for large N. To describe this algebra we first define the Bergman kernel

$$B(z, \bar{z}) = \sum_k s_k(z) \bar{s}_k(\bar{z}), \tag{2.7}$$

as well as

$$e(z, \bar{z}) = B(z, \bar{z}) e^{-K(z, \bar{z})} \tag{2.8}$$

and the Calabi function

$$\phi(z, \bar{z}; v, \bar{v}) = K(z, \bar{v}) + K(v, \bar{z}) - K(z, \bar{z}) - K(v, \bar{v}) \tag{2.9}$$

Note that e and ϕ are invariant under the Kähler transformation $K \to K + f + \bar{f}$. The Calabi function is well defined in a neighborhood of $M \times \overline{M}$. The $*$ product is then given by

$$
\begin{aligned}
(f * g)(z, \bar{z}) &= \int f(z, \bar{v}) g(v, \bar{z}) \frac{B(z, \bar{v}) B(v, \bar{z})}{B(z, \bar{z})} e^{-K(v, \bar{v})} d\mu_v \\
&= \int f(z, \bar{v}) g(v, \bar{z}) \frac{e(z, \bar{v}) e(v, \bar{z})}{e(z, \bar{z})} e^{\phi(z, \bar{z}; v, \bar{v})} d\mu_v
\end{aligned} \tag{2.10}
$$

Moreover, the trace of $\mathcal{O}_f$ is given by

$$\mathrm{tr}\mathcal{O}_f = \int f(z, \bar{z}) B(z, \bar{z}) e^{-K} d\mu = \int f(z, \bar{z}) e(z, \bar{z}) d\mu \tag{2.11}$$

The $*$ algebra reflects the operator algebra in that

$$\mathcal{O}_{f*g} = \mathcal{O}_f \mathcal{O}_g. \tag{2.12}$$

An differential expression for the $*$ product can be written as an expansion in $\frac{1}{N}$. To leading order one has

$$f * g = fg + \theta^{a\bar{b}} \partial_a f \partial_{\bar{b}} g + \cdots \qquad \theta^{a\bar{b}} = (\omega^{-1})^{a\bar{b}} + \cdots. \tag{2.13}$$

The corrections to these expression are given as a perturbative diagrammatic expansion in [13]. The star commutators of the complex coordinates themselves obey

$$[z^a, z^{\bar{b}}]_* = \theta^{a\bar{b}}. \tag{2.14}$$

2.2 Noncommutative configurations at large N

Now we wish to use this structure to describe nontrivial static configurations of N D0-branes on $CY_3 \times R^4$, in an expansion in $\frac{1}{N}$ as in [13]. We consider here only the leading order behavior. Suppressing for now the R^4 coordinates, N D0-brane configurations are described by 6 $N \times N$ matrices which will be denoted Φ^A, $A = 1, ..6,$. The matrices Φ^A can be viewed, with a suitable basis choice, as operators acting on the Hilbert space $\mathcal{H}$ of holomorphic sections of the line bundle $\mathcal{L}$ described in the preceding subsection.[3] As such we can associate to each matrix Φ^A the covariant symbol

$$X^A(z, \bar{z}) = \frac{\sum_{i,j=1}^{N} \bar{s}_i(\bar{z}) \Phi^A_{ij} s_j(z)}{\sum_{i=1}^{N} |s_i(z)|^2}. \tag{2.15}$$

According to (2.11) the matrix trace becomes the integral over CY_3, while from (2.13) the matrix product of Φ^A then becomes the star product of X^A on CY_3

$$\Phi^A_{ij} \Phi^B_{jk} \rightarrow X^A(z, \bar{z}) * X^B(z, \bar{z}). \tag{2.16}$$

The star product appearing here is the one associated to $\mathcal{L}$ as described above.

We do not expect that (2.14) is a solution for arbitrary θ, or equivalently, arbitrary ω. Rather the D0 equation of motion, or the requirement of unbroken supersymmetry, should constrain θ. To find these constraints to leading order for large CY_3, we may expand the D0 action around a point. Locally, the leading terms are given by the dimensional reduction of d=10 super Yang-Mills. Supersymmetry requires the familiar D and F-flatness conditions. The reduced action has a superpotential of the form $W \sim \epsilon_{abc} \operatorname{Tr} \Phi^a \Phi^b \Phi^c$. The F-flatness condition is the vanishing of

$$D_a W \sim \epsilon_{abc} [\Phi^b, \Phi^c], \tag{2.17}$$

where ϵ here is locally identified as the holomorphic three-form on CY_3. D-flatness is the condition

$$g_{a\bar{b}} \operatorname{Tr}[\Phi^a[T_i, \Phi^{\bar{b}}]] = 0. \tag{2.18}$$

The covariant symbol of (2.18) is

$$g_{a\bar{b}} \theta^{a\bar{b}} = 0. \tag{2.19}$$

This condition can be rephrased

$$d *_\omega J = 0, \quad J = ig_{a\bar{b}} dz^a dz^{\bar{b}}, \tag{2.20}$$

where $*_\omega$ is the Hodge dual constructed using ω as a Kahler form. Hence the physical J is harmonic with respect to ω. ω and θ will be determined in terms of the induced higher dimensional D-brane charges in the next subsection.

In fact the D-flatness condition (2.18) is a bit too strong. This condition arises from demanding the vanishing of the supersymmetry variation of the worldline fermions, which

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[3]We stress this line bundle $\mathcal{L}$ is in general different from the one associated to the physical Kahler form $J = ig_{a\bar{b}} dz^a dz^{\bar{b}}$ on CY_3.

are in the adjoint of $U(N)$. This worldline action has four linear and four nonlinear supersymmetries. The latter act in Goldstone mode as shifts of the $U(1)$ fermion. (2.18) arises from demanding an unbroken subgroup of the linearly realized supersymmetry. However it can also happen that there is an unbroken supersymmetry which is a linear combination of the original linear and nonlinear supersymmetries. This leads to a generalization of (2.18) with an arbitrary constant on the right hand side when T_i is the $U(1)$ generator. The covariant symbol of this is the generalization of (2.19)

$$g_{a\bar{b}}\theta^{a\bar{b}} = \text{constant}. \tag{2.21}$$

This weaker condition still implies (2.20), but relaxes the condition that θ and J are orthogonal.[4]

We will see in the next subsection that this D0 configuration describes a CY_3-wrapped D6 brane with worldvolume magnetic flux $F \sim \omega \sim \theta^{-1}$. From this point of view we might have expected ω to be harmonic which is not what we found above in (2.21).However the harmonic condition pertains only to leading order at small ω, while here ω is large. In this regime (2.21) in fact agrees with an analysis of the DBI action for the D6-brane. In the small θ, or large F limit, the D6-brane world volume BI action takes the form

$$S = \int \sqrt{\det(g + F)}$$
$$= \int \sqrt{\det F} \left[1 + \frac{1}{4}\text{Tr}(gF^{-1})^2 + \cdots \right] \tag{2.22}$$

When F is of type $(1,1)$, $\int \sqrt{\det F} = \int F^3$ is topological. We shall also use F^{-1} to denote the 2-form given by $(F^{-1})^{a\bar{b}}$ with indices lowered via the metric g. (2.21) implies that F^{-1} is harmonic with respect to g. It follows that $\sqrt{\det F} = const \times \sqrt{\det g}$, and $\text{Tr}(gF^{-1})^2 = \langle F^{-1}, F^{-1} \rangle$ is also constant. It is then easy to see that such F satisfies the equation of motion obtained from varying the second term in the expansion of (2.22).

In summary, to leading order for large ω and large CY_3, the D0 equations of motion imply that the Kähler form J is harmonic with respect to ω. Hence, (2.2), for so constrained θ, is to leading order a static non-abelian solution for N D0-branes.

2.3 Induced charges

For the noncommutative D0-brane configurations (2.14) the effective action (2.1) can be rewritten in terms of the covariant symbols X^A as

$$S = T_0 \int dt \int_{CY_3} d\mu \left\{ C_t^{(1)} + i\frac{\lambda}{2}[X^A, X^B]_*(C_{tAB}^{(3)} + C_t^{(1)}B_{AB}) \right. \tag{2.23}$$
$$\left. - \frac{\lambda^2}{8}[X^A, X^B]_*[X^C, X^D]_* \left(C_{tABCD}^{(5)} + C_{t[AB}^{(3)}B_{CD]} + \frac{1}{2}C_t^{(1)}B_{[AB}B_{CD]} \right) + \cdots \right\}_*$$

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[4]One can also see this directly from the supersymmetry transformation of the D0-brane world-line fermions [14], $\delta\psi = -iD_\tau X^A \gamma_A \epsilon + \frac{1}{2}[X^A, X^B]\gamma_{AB}\epsilon + \eta$, where ϵ and η are covariantly constant spinors on the CY_3.

where we replaced the trace by the integral over CY_3 via (2.11) and the subscript $*$ means replacing the ordinary products of the fields by the $*$ product.[5] Ordering issues will effect the higher order in θ corrections in (2.24).

In the small θ limit, the terms in (2.24) for which (2.14) sources Ramond-Ramond fields may be rewritten

$$\int dt \int_{CY_3} d\mu\, C_t^{(1)} = N \int dt C_t^{(1)}$$

$$\frac{1}{2} \int dt \int_{CY_3} d\mu\, \theta^{AB} C_{tAB}^{(3)} = q_I \int dt \int_{\alpha^I} C_t^{(3)}$$

$$\frac{1}{8} \int dt \int_{CY_3} d\mu\, \theta^{AB}\theta^{CD} C_{tABCD}^{(5)} = p^I \int dt \int_{\beta_I} C_t^{(5)}$$

$$\frac{1}{48} \int dt \int_{CY_3} d\mu\, \theta^{AB}\theta^{CD}\theta^{EF} C_{tABCDEF}^{(7)} = \int dt \int_{CY_3} C_t^{(7)}\,, \tag{2.24}$$

where the induced D4 and D2 charges are denoted by

$$p^I = \int_{\alpha^I} \omega, \qquad q_I = \int_{\beta_I} \frac{\omega^2}{2} \tag{2.25}$$

Note that the induced D6-brane charge is always 1 for this configuration. Hence we may interpret the D0-brane configuration $[\Phi^A, \Phi^B] = \theta^{AB}$ constructed from the deformation quantization with Kähler form ω as a single D6-brane wrapped on CY_3 with gauge field flux

$$F = \omega. \tag{2.26}$$

The latter has RR charges $Q = e^\omega \sqrt{Td(CY_3)}$ [15] which agrees with (2.25) at large N. The number of D0-branes contained in a D6-brane with nonzero F is given by the pairing[6] $(Q, \sqrt{Td(CY_3)}) = \int_{CY_3} e^\omega \cdot Td(CY_3)$ which is precisely the dimension N in (2.4). In this way the line bundle $\mathcal{L}$ we used to construct the Chan-Paton factors of the D0-branes is naturally interpreted as the $U(1)$ gauge bundle on the D6-brane. This is a generalization to Calabi-Yau spaces of the Myers effect [8] whereby higher brane charges are produced from collections of lower dimensional branes.

2.4 An example: the quintic

In this section we give an explicit example of a non-abelian configuration of 5 D0-branes carrying higher brane charges. Consider the quintic 3-fold

$$X_5 = \{(z^1)^5 + (z^2)^5 + (z^3)^5 + (z^4)^5 + (z^5)^5 = 0\} \subset \mathbf{P}^4$$

with the Kähler form ω being the curvature of the $\mathcal{O}(1)$ bundle. We take ω as induced from the Fubini-Study metric on $\mathbf{P}^4$, i.e.

$$\omega = \frac{i}{2\pi} \partial\bar{\partial} \ln\left(\sum_{a=1}^{5} |z^a|^2\right). \tag{2.27}$$

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[5] We have used $e(z,\bar{z}) = 1$ for large N. Note also that for large N, e^{-K} will be supported near a few points on CY_3 corresponding to the position of the N D0-branes.

[6] This is also the index of the Dirac operator $\not{D}$ on the Chan-Paton bundle, which measures the chiral zero modes of open strings stretched between the D6 and D0-branes.

We can write the Kähler potential for ω as $K = \ln(\sum_a |z^a|^2)$. $\mathcal{L} = \mathcal{O}_M(1)$ has $N = 5$ sections, corresponding to monomials $z^1, z^2, \cdots, z^5$. The Bergman kernel is

$$B(z, \bar{z}) = c \sum_a z^a \bar{z}^a \tag{2.28}$$

The z^a's are, of course, defined only up to a local holomorphic rescaling, i.e. after picking a local trivialization of $\mathcal{L}$. The normalization can be determined from

$$\int B e^{-K} d\mu = c \cdot \text{vol}(X_5) = 5 \tag{2.29}$$

Here $\text{vol}(X_5) = \int \frac{\omega^3}{6} = 5/6$, so we get $c = 6$. The Calabi function is

$$\phi(z, \bar{z}; v, \bar{v}) = \ln \frac{|\sum_{a=1}^5 z^a \bar{v}^a|^2}{(\sum_b |z^b|^2)(\sum_c |v^c|^2)} \tag{2.30}$$

The star product is given by (2.10)

$$(f * g)(z, \bar{z}) = \int d\mu_v f(z, \bar{v}) g(v, \bar{z}) \frac{6 |\sum_{a=1}^5 z^a \bar{v}^a|^2}{(\sum_b |z^b|^2)(\sum_c |v^c|^2)} \tag{2.31}$$

Given a suitable function $f(z, \bar{z})$, the associated operator $\mathcal{O}_f$ acts on a section $s(z)$ as

$$\begin{aligned}
(\mathcal{O}_f s)(z) &= \int d\mu_v f(z, \bar{v}) B(z, \bar{v}) s(v) e^{-K(v, \bar{v})} \\
&= \int d\mu_v f(z, \bar{v}) s(v) \frac{6 \sum_a z^a \bar{v}^a}{\sum_b |v^b|^2}
\end{aligned} \tag{2.32}$$

In particular, for functions

$$f_{ab}(z, \bar{z}) = \frac{z^a \bar{z}^b}{\sum_{c=1}^5 |z^c|^2} \tag{2.33}$$

The associated operators $\mathcal{O}_{ab}$ acts on section z^k as

$$\mathcal{O}_{ab} \cdot z^c = \int d\mu_v \frac{6 z^a \bar{v}^b v^c}{\sum_d |v^d|^2} = \frac{6}{5} \left(\int d\mu_v \right) \delta_{bc} z^a = \delta_{bc} z^a \tag{2.34}$$

In other words, written as 5×5 matrices, we have

$$(\mathcal{O}_{ab})_{mn} = \delta_{am} \delta_{bn} \tag{2.35}$$

In general, a 5×5 matrix A acting on $H^0(X_5; \mathcal{L})$ is associated with the function

$$f_A(z, \bar{z}) = \frac{\sum_{a,b} A_{ab} z^a \bar{z}^b}{\sum_c |z^c|^2} \tag{2.36}$$

on X_5. These are nothing but the covariant symbols (2.5).

We can choose a set of local coordinates near say the point $[z^1, z^2, z^3, z^4, z^5] = [0, 0, 0, -1, 1]$ to be, in an obvious notation, $f_{15}, f_{25}, f_{35}, f_{51}, f_{52}, f_{53}$. Fixing $z^5 = 1$, we define

$$x^a = f_{a5} = \frac{1}{2} z^a + \mathcal{O}(|z|^3), \quad y^a = f_{5a} = \frac{1}{2} \bar{z}^a + \mathcal{O}(|z|^3), \quad i = 1, 2, 3. \tag{2.37}$$

Their commutation relations are

$$[x^a, x^b]_* = [y^a, y^b]_* = 0,$$
$$[x^a, y^b]_* = f_{ab}(x, y) - \delta_{ab} f_{55}(x, y) = -\frac{1}{2}\delta_{ab} + 2x^a y^b + \cdots \tag{2.38}$$

where the omitted terms are of quadratic or higher order in x and y.

Now we may use this to construct a non-abelian configuration of 5 D0-branes which grows higher brane charges. We simply take the D0-brane matrix Φ^i ($i = 1, \cdots, 6$), in the coordinates (2.37), to be the one whose covariant symbol is x^a or y^a, namely :

$$(\Phi^a)_{mn} = \delta_{am}\delta_{5n}, \quad (\Phi^{a+3})_{mn} = \delta_{5m}\delta_{an}. \tag{2.39}$$

(2.39) does not obey the equations of motion (2.21) for static solutions. Indeed our construction did not involve the physical metric $g_{a\bar{b}}$. Hence the configuration will move under time evolution. Finding exact static solutions is a more difficult problem which would require knowing the Ricci-flat metric on the quintic.

It is straightforward in principle but tedious in practice to generalize the above construction to ω being n times the unit integral Kähler form, correspondingly $\mathcal{L}$ is the $\mathcal{O}(n)$ bundle. In the case $n \leq 4$, the sections simply correspond to degree n monomials of the z^a's. When $n \geq 5$, there are additional relations between the monomials.

3. Superconformal quantum mechanics of D0-branes in $AdS_2 \times S^2 \times CY_3$

Near the horizon of a supersymmetric type II black hole, spacetime approaches an $AdS_2 \times S^2 \times CY_3$ attractor geometry. In this region the RR fluxes are important in the D0 dynamics, and the vector moduli of CY_3 approach fixed attractor values which are determined by the black hole charges. The attractor equations governing the moduli X^Λ are [16][17]

$$p^\Lambda = \text{Re}[CX^\Lambda], \quad q_\Lambda = \text{Re}[CF_\Lambda], \tag{3.1}$$

where $\Lambda = 0, 1 \cdots b_2$, p_Λ (q_Λ) are the magnetic (electric) charges of the black hole, F_Λ are the periods of CY_3 and C may be set to one by a gauge transformation. In this section will consider D0-branes in these attractor geometries.

3.1 Landau levels and the noncommutative horizon

A single D0-brane couples to the RR gauge field potential via the first term in (2.1)

$$T_0 \int dt C_t^{(1)}, \tag{3.2}$$

where $C^{(1)}$ is sourced by D6-branes. In the attractor geometry arising from p^0 D6-branes (as well as other branes), there is a nonzero two-form magnetic field strength

$$F^{(2)} = p^0 \epsilon_{S2}, \tag{3.3}$$

where the unit volume form ϵ_{S2} integrates to 4π over S^2. This implies, as for an electron in a magnetic field, that at low energies the coordinates of a single D0 become noncommuting

$$[\Phi^A, \Phi^B] = \left((F^{(2)})^{-1}\right)^{AB}, \tag{3.4}$$

and that the low energy configurations have S^2 wave functions described by one of the $p^0 + 1$ lowest Landau levels.[7] It can be seen[8] that for an appropriate AdS_2 trajectory, as described in [6], these lowest Landau levels preserve one-half, or a total of 4, near-horizon supersymmetries. Hence a single D0 in an attractor geometry has $p^0 + 1$ degenerate supersymmetric ground states

$$\Omega(1) = p^0 + 1. \tag{3.5}$$

These ground states can be pictured as lowest Landau levels tiling the horizon of the black hole.

The above result can be generalized to the case of a general probe brane with four-dimensional magnetic-electric charge vector (u^Λ, v_Λ) in an attractor geometry with fluxes (p^Λ, q_Λ). Symplectic invariance fixes the degeneracy of the lowest Landau level to be

$$e^{S(u,v)} = |p^\Lambda v_\Lambda - q_\Lambda u^\Lambda|. \tag{3.6}$$

By "probe" brane here we mean that it can be treated as a single object with no internal moduli in the internal space or degeneracies (beyond those implied by supersymmetry). This will not be the case for branes wrapping curves of high degrees in CY_3, which will have additional degeneracies.

3.2 Conformal quantum mechanics

We will be interested in a D0-brane moving in the near horizon $AdS_2 \times S^2 \times CY_3$ geometry of D0-D4 black hole, with magnetic-electric charges $(p^0 = 0, p^A, q_0, q_A = 0)$. The bosonic part of the world-line action takes the form

$$-m \int d\tau + q \int A \tag{3.7}$$

where $q = m$ as the D0-brane is "extremal" in this background. The radius R of $AdS_2 \times S^2$ is equal to the graviphoton charge Q,

$$R = Q = (4Dq_0)^{\frac{1}{4}} l_4 = \frac{1}{2} g_s \sqrt{\frac{D\alpha'}{q_0}} \tag{3.8}$$

where l_4 is the four-dimensional Planck length.

The near horizon geometry $AdS_2 \times S^2$ of an $\mathcal{N} = 2$ black hole with general magnetic-electric charges (p^I, q_I) has gauge field strengths $\hat{F}^I, \hat{G}_I$ satisfying

$$p^I = \frac{1}{4\pi} \int_{S^2} \hat{F}^I, \quad q_I = \frac{1}{4\pi} \int_{S^2} \hat{G}_I, \quad \hat{G}_I^+ = \mathcal{N}_{IJ} \hat{F}^{+J}. \tag{3.9}$$

---

[7]The number of lowest Landau levels is $p^0 + 1$ instead of p^0 due to the curvature of the S^2. It also follows from the Riemann-Roch formula.

[8]For example by lifting to the M-theory picture or looking at the reduced quantum mechanics problem.

where $\mathcal{N}_{IJ}$ is the gauge coupling matrix. These relations can be solved by

$$\hat{F}^I = p^I \omega_{S^2} + f^I \omega_{AdS_2},$$
$$\hat{G}_I = q_I \omega_{S^2} + g_I \omega_{AdS_2},$$
$$CF_I = q_I + ig_I, \quad CX^I = p^I + if^I. \tag{3.10}$$

A superparticle of charges (u^I, v_I) coupled to the gauge fields via

$$\int_\Sigma u^I \hat{G}_I - v_I \hat{F}^I = q_e R \int \frac{dt}{\sigma} + q_m R \int \cos\theta d\phi \tag{3.11}$$

where Σ is an auxiliary surface whose boundary is the world line. We have

$$m = |Z| = |u^I F_I - v_I X^I|,$$
$$q_e = \text{Im}(CZ),$$
$$q_m = \text{Re}(CZ) = (p^I v_I - q_I u^I). \tag{3.12}$$

where we have normalized $|C| = R$.

In the case $q_e = m$, one can rewrite the action in Poincaré coordinate as

$$-m \int dt \sqrt{R^2 \left[\sigma^{-2}(1 - \dot{\sigma}^2) - \dot{\theta}^2 - \sin^2\theta \dot{\phi}^2 \right] - 2 g_{a\bar{b}} \dot{z}^a \dot{\bar{z}}^{\bar{b}}} + mR \int dt \frac{1}{\sigma} \tag{3.13}$$

where $g_{a\bar{b}}$ is the metric on the CY. Expanding this action to quadratic order in the time derivatives and replacing σ with

$$\xi = \sqrt{\sigma}, \tag{3.14}$$

we can write the action as

$$m \int dt \left[2R\dot{\xi}^2 + \frac{R}{2}\xi^2 \left(\dot{\theta}^2 + \sin^2\theta \dot{\phi}^2 \right) + \frac{\xi^2}{R} g_{a\bar{b}} \dot{z}^a \dot{\bar{z}}^{\bar{b}} \right] \tag{3.15}$$

In terms of the conjugate momenta

$$P_\xi = 4mR\dot{\xi}, \quad P_\theta = mR\xi^2 \dot{\theta}, \quad P_\phi = mR\xi^2 \sin^2\theta \dot{\phi}, \quad P_a = \frac{m}{R}\xi^2 g_{a\bar{b}} \dot{\bar{z}}^{\bar{b}} \tag{3.16}$$

we have the hamiltonian

$$H = \frac{P_\xi^2}{8mR} + \frac{1}{2mR\xi^2} \left(P_\theta^2 + \frac{P_\phi^2}{\sin^2\theta} \right) + \frac{R}{m\xi^2} P_a g^{a\bar{b}} P_{\bar{b}} \tag{3.17}$$

A homothety of the full metric appearing in (3.17) is

$$D = \xi \frac{\partial}{\partial \xi} \tag{3.18}$$

The corresponding operator in the conformal quantum mechanics is given by

$$D = \frac{1}{2}(\xi P_\xi + P_\xi \xi) \tag{3.19}$$

and the special conformal generator is

$$K = 2mR\xi^2 \tag{3.20}$$

It is easy to check that H, D, K indeed obey the commutation relations of $SL(2,R)$

$$[D, H] = 2iH, \quad [D, K] = -2iK, \quad [H, K] = -iD \tag{3.21}$$

We can in fact write down the exact Hamiltonian without making the linear approximation in (3.13). Writing in terms of $\sigma = \xi^2$

$$f = 1 - \dot{\sigma}^2 - \sigma^2 \left(\dot{\theta}^2 + \sin^2\theta \dot{\phi}^2 + \frac{1}{R^2} g_{ij} \dot{y}^i \dot{y}^j \right) \tag{3.22}$$

we have

$$P_\sigma = mR\frac{\dot{\sigma}}{\sigma\sqrt{f}}, \quad P_\theta = mR\frac{\sigma\dot{\theta}}{\sqrt{f}}, \quad P_\phi = mR\frac{\sigma\sin^2\theta\dot{\phi}}{\sqrt{f}}, \quad P_i = \frac{m}{R}\frac{\sigma g_{ij}\dot{y}^j}{\sqrt{f}}. \tag{3.23}$$

The Hamiltonian is then

$$H = \sqrt{\frac{m^2R^2}{\sigma^2} + P_\sigma^2 + \frac{1}{\sigma^2}(P_\theta^2 + \frac{1}{\sin^2\theta}P_\phi^2 + R^2 P_i g^{ij} P_j)} - \frac{mR}{\sigma} \tag{3.24}$$

For more general (q_e, q_m), corresponding to the probe D0-brane having charge not aligned with the black hole, the RR 1-form potential is

$$A = q_e R \frac{dt}{\sigma} + q_m R \cos\theta d\phi \tag{3.25}$$

Now we have

$$P_\phi = \frac{mR\sigma \sin^2\theta\dot{\phi}}{\sqrt{f}} + q_m R \cos\theta \tag{3.26}$$

The full Hamiltonian is then

$$H = \sqrt{\frac{m^2R^2}{\sigma^2} + P_\sigma^2 + \frac{1}{\sigma^2}\left[P_\theta^2 + \frac{1}{\sin^2\theta}(P_\phi - q_m R \cos\theta)^2 + R^2 P_i g^{ij} P_j \right]} - \frac{q_e R}{\sigma} \tag{3.27}$$

The dilatation operator is as usual,

$$D = \sigma P_\sigma + P_\sigma \sigma. \tag{3.28}$$

3.3 Superconformal mechanics on $AdS_2 \times S^2$

As a warm up to the full $AdS_2 \times S^2 \times CY_3$ case, in this subsection we construct the superconformal zerobrane quantum mechanics of $AdS_2 \times S^2$ without the CY_3. This extends the construction of [18, 19] by the addition of a magnetic field coupling to the zerobrane.

The collective coordinates of the zerobrane are the bosonic part of an $\mathcal{N} = 4$ multiplet. In $\mathcal{N} = 1$ language it consists of bosonic superfields $\mathbf{X}^i$ and fermionic superfield Ψ, where

$$\mathbf{X}^i = x^i - i\theta\lambda^i, \quad \mathbf{\Psi} = i\psi + i\theta b. \tag{3.29}$$

A general $\mathcal{N} = 1$ supersymmetric action including coupling to magnetic field and scalar potential takes the form [20]

$$S = \int dtd\theta \left[\frac{i}{2}g_{ij}(\mathbf{X})D\mathbf{X}^i\dot{\mathbf{X}}^j + \frac{1}{6}c_{ijk}(\mathbf{X})D\mathbf{X}^iD\mathbf{X}^jD\mathbf{X}^k \right.$$
$$\left. +iA_i(\mathbf{X})D\mathbf{X}^i - \frac{1}{2}h(\mathbf{X})\Psi D\Psi + is(\mathbf{X})\Psi \right] \tag{3.30}$$

$\mathcal{N} = 4$ supersymmetry requires g_{ij}, c_{ijk}, h to be related to a function $L(\mathbf{X})$ via [18]

$$g_{ij} = \partial_i\partial_j L + \epsilon_i{}^{mk}\epsilon_{jm}{}^l\partial_k\partial_l L,$$
$$c_{ijk} = \frac{1}{2}\epsilon^{pqr}\epsilon_p{}^l{}_i\epsilon_q{}^m{}_j\epsilon_r{}^n{}_k\partial_{lmn}L,$$
$$h = \delta^{ij}\partial_i\partial_j L. \tag{3.31}$$

As described in the previous subsection, the sigma model metric is obtained from the non-relativistic limit of a particle in the Poincaré patch of $AdS_2 \times S^2$, given by

$$ds^2 = \frac{dx^k dx^k}{|\vec{x}|} \tag{3.32}$$

A natural choice of L which produces the metric (3.32) via (3.31) is

$$L(X) = \frac{1}{2}|\vec{X}| \tag{3.33}$$

It follows that

$$c_{ijk} = 0, \quad h(X) = \frac{1}{|\vec{X}|}. \tag{3.34}$$

The terms involving the auxiliary field b in (3.30) is

$$\frac{1}{2}h(x)b^2 - s(x)b. \tag{3.35}$$

Integrating out b yields the scalar potential

$$V(x) = \frac{s(x)^2}{2h(x)}. \tag{3.36}$$

Conformal invariance requires $V(x) \propto 1/|\vec{x}|$, so we have

$$s(X) = \frac{B}{|\vec{X}|} \tag{3.37}$$

for some constant B. With (3.32),(3.34), and (3.37), the action (3.30) can be written as

$$S = \int dtd\theta \left[\frac{1}{2|\vec{\mathbf{X}}|}D\mathbf{X}^i\dot{\mathbf{X}}^i + iA_i(\mathbf{X})D\mathbf{X}^i - \frac{1}{2|\vec{\mathbf{X}}|}\Psi D\Psi + \frac{iB}{|\vec{\mathbf{X}}|}\Psi \right] \tag{3.38}$$

The 3 additional supercharges act on the $\mathcal{N} = 1$ superfields as

$$Q_i\mathbf{X}^j = \epsilon_i{}^j{}_kD\mathbf{X}^k + \delta_i^j\Psi,$$
$$Q_i\Psi = i\delta_{ij}\dot{\mathbf{X}}^j. \tag{3.39}$$

The action (3.38) is invariant under Q_i's when $A_i(X)$ is the vector potential corresponding to magnetic field $F_{ij} = B\epsilon_{ijk}X^k/|\vec{X}|^3$, i.e. that of a monopole of charge B at the origin. We see that $\mathcal{N} = 4$ supersymmetry determines the potential (3.36) in terms of the magnetic field B on the S^2. This is expected from the bosonic action one obtains by expanding the Born-Infeld action near the charged geodesic of the zerobrane, as in section 3.2.

One of the supercharges takes the form

$$Q = -i\lambda^i(\nabla_i + A_i) + \psi s(\vec{x})$$
$$= -i\lambda^i(\partial_i - \frac{x^i}{2|\vec{x}|^2} + A_i) + \psi\frac{B}{|\vec{x}|} \quad (3.40)$$

where ∇ is the spin connection. The corresponding special supercharge is

$$S = \lambda^i D_i = \lambda^i \frac{x^i}{|\vec{x}|} \quad (3.41)$$

3.4 Including the CY_3 factor

In this section we give the full $SU(1,1|2)$ quantum mechanics for a D0 brane in an attractor geometry.

The metric for D0-brane quantum mechanics on $AdS_2 \times S^2 \times CY_3$ is

$$ds^2 = 4Qd\xi^2 + \xi^2\left(Qd\Omega_2^2 + \frac{2}{Q}g_{a\bar{b}}dy^a dy^{\bar{b}}\right)$$
$$= \frac{1}{|\vec{x}|}\sum_{k=1}^{3} dx^k dx^k + \frac{2|\vec{x}|}{Q^2}g_{a\bar{b}}dz^a dz^{\bar{b}}. \quad (3.42)$$

We will work in the coordinate system $(\xi = \sqrt{\sigma}, \theta, \phi, z^a, z^{\bar{a}})$. There are four supercharges $Q_\alpha, \bar{Q}_\alpha$ and four special supercharges $S_\alpha, \bar{S}_\alpha$, where $\alpha = 1, 2$ is a doublet index under $SU(2)_R \subset SU(1,1|2)$. The fermions are $\lambda_\alpha, \bar{\lambda}_\alpha$ which are roughly the λ^i, ψ appeared in the SCQM for D0-brane on $AdS_2 \times S^2$, and $\eta_\alpha^a, \bar{\eta}_\alpha^{\bar{a}}$ which are roughly the superpartners of $z^a, z^{\bar{a}}$, $a = 1, 2, 3$. The $SU(2)$ doublet index will be raised and lowered using $\epsilon_{\alpha\beta}$, for example, $\lambda^\alpha = \lambda_\beta\epsilon^{\beta\alpha}$, $\lambda_\alpha = \epsilon_{\alpha\beta}\lambda^\beta$. Let $\hat{L}_i^{S^2}$ be the angular momentum on the S^2. We will be free to trade the vector index i with a symmetric pair of spinor indices $(\alpha\beta)$. We shall also define the $SU(2)_R$ generators on λ and η:

$$\hat{L}_{\alpha\beta}^\lambda = \lambda_{(\alpha}\bar{\lambda}_{\beta)},$$
$$\hat{L}_{\alpha\beta}^\eta = g_{a\bar{b}}^{CY}\eta_{(\alpha}^a\bar{\eta}_{\beta)}^{\bar{b}}. \quad (3.43)$$

The $SU(2)_{\text{Right}}$ generators are

$$T_{\alpha\beta} = \hat{L}_{\alpha\beta}^{S^2} + L_{\alpha\beta}^\lambda + L_{\alpha\beta}^\eta \quad (3.44)$$

The supercharges are then given by

$$Q_\alpha = Q^{-\frac{1}{2}}\left[\frac{1}{2}\lambda_\alpha\hat{P}_\xi - \frac{i}{\xi}(L_{\alpha\beta}^{S^2} + L_{\alpha\beta}^\eta)\lambda^\beta + \frac{i}{4\xi}\bar{\lambda}_\alpha\lambda^2 + \frac{i}{4\xi}\lambda_\alpha\right] + \frac{\sqrt{2Q}}{\xi}\eta_\alpha^a\hat{P}_a^{CY},$$

$$\bar{Q}_\alpha = Q^{-\frac{1}{2}}\left[\frac{1}{2}\bar{\lambda}_\alpha\hat{P}_\xi - \frac{i}{\xi}(L_{\alpha\beta}^{S^2} + L_{\alpha\beta}^\eta)\bar{\lambda}^\beta - \frac{i}{4\xi}\bar{\lambda}^2\lambda_\alpha - \frac{i}{4\xi}\bar{\lambda}_\alpha\right] + \frac{\sqrt{2Q}}{\xi}\bar{\eta}_\alpha^{\bar{a}}\hat{P}_{\bar{a}}^{CY},$$

$$S_\alpha = 2Q^{\frac{1}{2}}\xi\lambda_\alpha, \quad \bar{S}_\alpha = 2Q^{\frac{1}{2}}\xi\bar{\lambda}_\alpha. \quad (3.45)$$

where $\lambda^2 \equiv \lambda_\beta \lambda^\beta,\ \bar{\lambda}^2 \equiv \bar{\lambda}_\beta \bar{\lambda}^\beta$. They satisfy commutation relations

$$\begin{aligned}
\{Q_\alpha, Q_\beta\} &= 0, & \{Q_\alpha, \bar{Q}_\beta\} &= 2\epsilon_{\alpha\beta}H, \\
\{S_\alpha, \bar{S}_\beta\} &= 2\epsilon_{\alpha\beta}K, & \{Q_\alpha, \bar{S}_\beta\} &= \epsilon_{\alpha\beta}D - 2iT_{\alpha\beta}.
\end{aligned} \tag{3.46}$$

The wave functions on the CY can be identified with forms as usual. The doublets $\eta_\alpha^a P_a, \bar{\eta}_\alpha^{\bar{a}} P_{\bar{a}}$ are then identified with differential operators on the CY as

$$(\,\eta_1^a P_a \eta_2^a P_a\,) \to (\quad \partial \quad \bar{\partial}^*\,), \qquad (\,\bar{\eta}_1^{\bar{a}} P_{\bar{a}} \bar{\eta}_2^{\bar{a}} P_{\bar{a}}\,) \to (\,\bar{\partial} \quad\quad -\partial^*\quad) \tag{3.47}$$

$\hat{L}_{\alpha\beta}^\eta$ are identified with the generators of Lefschetz action on the forms. Note that in (3.45) the angular momentum of the D0-brane on the S^2 is curiously twisted by the Lefschetz weight of the state. In addition to $SU(2)_R$, we have an additional $SU(2)_L$ R-symmetry which rotates $Q, \bar{Q}$ as a doublet. The $SU(2)_L$ generators are given by

$$\begin{aligned}
J_L^+ &= \lambda^2 + P_{\mathcal{H}^\perp} \frac{1}{\Delta} \bar{\partial}\partial^*, \\
J_L^- &= \bar{\lambda}^2 + P_{\mathcal{H}^\perp} \frac{1}{\Delta} \partial\bar{\partial}^*, \\
J_L^3 &= \frac{1}{2}\lambda\bar{\lambda} - \frac{1}{2} + P_{\mathcal{H}^\perp} \frac{1}{2\Delta} (\partial^*\partial - \bar{\partial}^*\bar{\partial}).
\end{aligned} \tag{3.48}$$

where Δ is the Laplacian, $P_{\mathcal{H}^\perp}$ is the projection onto the orthogonal of the harmonic space. Note that the RHS of (3.48) is well defined since Δ^{-1} is bounded on $\mathcal{H}^\perp$ for compact CY_3. Together with the $SU(1,1|2)$ constructed above we have the full $D(2,1;0)$ superconformal algebra.

In the $SU(2)_R \times SU(2)_L$ doublet notation, we shall write

$$Q_1 = Q^{++}, \quad Q_2 = Q^{-+}, \quad \bar{Q}_1 = Q^{+-}, \quad \bar{Q}_2 = Q^{--}. \tag{3.49}$$

Now

$$\begin{aligned}
Q^{\pm+} &= Q^{-\frac{1}{2}} \left[\frac{1}{2}\lambda^{\pm+}\hat{P}_\xi - \frac{i}{\xi}(L^{S2\pm}{}_\alpha + L^{\eta\pm}{}_\alpha)\lambda^{\alpha+} + \frac{i}{4\xi}\{\lambda^{\pm-}, \lambda^{-+}\lambda^{++}\} \right] + \frac{\sqrt{2Q}}{\xi}\eta^{\pm+} \cdot \hat{P}^{CY}, \\
Q^{\pm-} &= Q^{-\frac{1}{2}} \left[\frac{1}{2}\lambda^{\pm-}\hat{P}_\xi - \frac{i}{\xi}(L^{S2\pm}{}_\alpha + L^{\eta\pm}{}_\alpha)\lambda^{\alpha-} - \frac{i}{4\xi}\{\lambda^{\pm+}, \lambda^{--}\lambda^{+-}\} \right] + \frac{\sqrt{2Q}}{\xi}\eta^{\pm-} \cdot \hat{P}^{CY}, \\
S^{\pm\pm} &= 2Q^{\frac{1}{2}}\xi\lambda^{\pm\pm}.
\end{aligned} \tag{3.50}$$

3.5 Chiral primaries

In this section we describe the chiral primaries states of the superconformal quantum mechanics. Related discussions appear in [21], [22].

Let us define

$$G_{\pm\frac{1}{2}}^{\alpha A} = \frac{1}{\sqrt{2}} \left(Q^{\alpha A} \mp i S^{\alpha A} \right). \tag{3.51}$$

where $\alpha, A = \pm$. We shall look for chiral primaries, which are states annihilated by the supercharges

$$G_{\pm\frac{1}{2}}^{++}, \quad G_{\pm\frac{1}{2}}^{+-}, \quad G_{\frac{1}{2}}^{-+}, \quad G_{\frac{1}{2}}^{--}. \tag{3.52}$$

Such a state $|0\rangle$ must be annihilated by S^{+A}, hence λ^{+A}, for $A = \pm$. We can demand that the CY_3 part of $|0\rangle$ is a harmonic form, i.e. it is annihilated by $\eta^{\pm\pm} \cdot \hat{P}^{CY}$. With $T^{++}|0\rangle = 0$, i.e. $|0\rangle$ has the highest $SU(2)_R$ weight, the condition $G^{+A}_{\pm\frac{1}{2}}|0\rangle = 0$ is automatically satisfied. Requiring $G^{-\pm}_{\frac{1}{2}}|0\rangle = 0$ now gives

$$\left(-\frac{i}{2}\lambda^{-+}\partial_\xi + \frac{i}{\xi}(L^{S^2} + L^\eta)^{-+}\lambda^{-+} + \frac{i}{4\xi}\lambda^{-+} - 2iQ\xi\lambda^{-+} \right)|0\rangle = 0 \,,$$

$$\left(-\frac{i}{2}\lambda^{--}\partial_\xi + \frac{i}{\xi}(L^{S^2} + L^\eta)^{-+}\lambda^{--} + \frac{i}{4\xi}\lambda^{--} - 2iQ\xi\lambda^{--} \right)|0\rangle = 0. \qquad (3.53)$$

The third $SU(2)_R$ generator is $J^3_R = T^{-+} = (L^{S^2} + L^\eta)^{-+} + \frac{1}{2}$ when acting on $|0\rangle$. We conclude that a general chiral primary state takes the form

$$|0\rangle \sim e^{-2Q\xi^2}\xi^{2j_R - \frac{1}{2}}|j_R, J^3_R = j_R; L^{S^2}, L^\lambda = \tfrac{1}{2}, \omega_{CY}\rangle \qquad (3.54)$$

where ω_{CY} is a primitive harmonic form on the CY_3. For example, if we identify $(L^\eta)^{++} = i_J$ where J is the Kähler form, the primitive harmonic forms are 1 (in the spin 3/2 representation), $h^{11} - 1$ harmonic 2-forms ω satisfying $\langle J, \omega \rangle = 0$ (in the spin 1/2 representation), and $2(h^{21} + 1)$ harmonic 3-forms (spin 0).

Now let us add magnetic field $F_{ij} = B\epsilon_{ijk}x^k/|x|^3$. If we replace the standard derivatives by gauge covariant derivatives, the angular momenta $L^{S^2}_i$ gets modified to $\tilde{L}^B_i$, which satisfy commutation relations

$$[\tilde{L}^B_i, \tilde{L}^B_j] = i\epsilon_{ijk}\left(\tilde{L}^B_k + B\frac{x^k}{|x|} \right) \qquad (3.55)$$

We can further define

$$L^B_k = \tilde{L}^B_k - \frac{Bx^k}{2|x|} \qquad (3.56)$$

so that L^B_k now satisfy the standard $SU(2)$ algebra. The supercharges are given by the expression (3.45) with L^{S^2} replaced by L^B. Note that $(L^B)^2 = (\tilde{L}^B)^2 + B^2/4$. The chiral primaries are still given by (3.54), with L^{S^2} replaced by L^B. The only difference is that we must restrict the spin $L^B = \frac{B}{2}, \frac{B}{2} + 1, \frac{B}{2} + 2, \cdots$.

We have another $U(1)$ R-symmetry that rotates the λ, η's and $\bar{\lambda}, \bar{\eta}$'s with opposite phases. This $U(1)$ generator is

$$\tilde{J} = \frac{1}{2}\lambda_\alpha\bar{\lambda}^\alpha + \frac{1}{2}g_{a\bar{b}}\eta^a_\alpha\bar{\eta}^{\bar{b}\alpha} - 1 \qquad (3.57)$$

The chiral primaries states (3.54) have $\tilde{J} = (p - q)/2$ if ω_{CY} is a (p,q)-form. $\tilde{J}$ doesn't commute with $J^\pm_L$. We can however take the linear combination $J_0 \equiv \tilde{J} - J^3_L$, which commutes with $SU(2)_L$. The corresponding $U(1)$ symmetry will be denoted $U(1)_0$.

We can assemble the chiral primaries into representations of $SU(2)_L \times SU(2)_R \times U(1)_0$, where $SU(2)_R$ is the R-symmetry appearing in $SU(1,1|2)$. The spectrum of the chiral primaries is

$$\bigoplus_{n=0}^{\infty} \left[\left(0, \frac{B}{2} + n + \frac{1}{2}, \frac{3}{2} \right) \oplus h^{21}\left(0, \frac{B}{2} + n + \frac{1}{2}, \frac{1}{2} \right) \right.$$

$$\oplus h^{21} \left(0, \frac{B}{2} + n + \frac{1}{2}, -\frac{1}{2}\right) \oplus \left(0, \frac{B}{2} + n + \frac{1}{2}, -\frac{3}{2}\right)$$

$$\oplus \left(0, \left(\frac{B}{2} + n\right) \otimes \left(\frac{3}{2}\right) + \frac{1}{2}, 0\right) \oplus (h^{11} - 1) \left(0, \left(\frac{B}{2} + n\right) \otimes \left(\frac{1}{2}\right) + \frac{1}{2}, 0\right) \right] \quad (3.58)$$

where we used the shorthand notation $(n) \otimes (m)$ for all the $SU(2)$ highest weights appearing in the tensor product $(n) \otimes (m)$. In the case $B = 0$, we can write the chiral primaries labeled by $SU(2)_L \times SU(2)_R$ representations as

$$\bigoplus_{n=0}^{\infty} \left[h^{odd} \left(0, n + \frac{1}{2}\right) \oplus h^{even}(0, n + 1) \right] \quad (3.59)$$

The chiral primaries sit in short representations of $D(2, 1; 0)$, denoted by $(j_L, j_R)_S$, which has spin contents[23] (see also the appendix)

$$(j_L, j_R)_S \rightarrow (j_L, j_R)_0 + \left(j_L + \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{1}{2}} + \left(j_L - \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{1}{2}} + (j_L, j_R - 1)_1 \quad (3.60)$$

where the subscripts label the difference of the L_0 value of the corresponding $SU(2) \times SU(2)$ representation from that of the chiral primary. The chiral primaries in our SCQM have $j_L = 0$, so the relevant short representation spin content is

$$(0, j_R)_S \rightarrow (0, j_R)_0 + \left(\frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{1}{2}} + (0, j_R - 1)_1 \quad (3.61)$$

States in a short multiplet are obtained by acting $G_{-1/2}^{\pm\pm}$ on the chiral primary. We have $L_0 = j_R$ for the chiral primary state, and with each action of $G_{-1/2}$, L_0 increases by $1/2$.

3.6 An index

Information about the chiral primaries can be summarized by an index which we now discuss.

Other, non-chiral-primary, states in the superconformal quantum mechanics form long representations $(j_L, j_R)_{long}$, which has the spin content of four short multiplets

$$(j_L, j_R)_{long} \rightarrow (j_L, j_R)_S + \left(j_L - \frac{1}{2}, j_R + \frac{1}{2}\right)_S + \left(j_L + \frac{1}{2}, j_R + \frac{1}{2}\right)_S + (j_L, j_R + 1)_S \quad (3.62)$$

A general long multiplet satisfies the BPS bound $L_0 \geq j_R$. A generic generating function $\mathrm{Tr}\, y^{J_L^3} z^{J_R^3} q^{L_0}$ receives contribution from both short and long multiplets. We can, however, construct the index $\mathrm{Tr}\,(-1)^{2J_L^3} y^{L_0 + J_L^3} z^{L_0 + J_R^3}$ which vanishes for long multiplets but doesn't vanish for short multiplets. Evaluating the trace over short multiplets gives

$$\mathrm{Tr}_{(j_L, j_R)_S}(-1)^{2J_L^3} y^{L_0 + J_L^3} z^{L_0 + J_R^3} = (-1)^{2j_L} y^{j_R - j_L} \frac{(1 - yz)(1 - y^{2j_L + 1})}{1 - y} \quad (3.63)$$

We are free to insert w^{J_0} into the trace to get a slightly refined index. In the case $B = 0$, it is straightforward to calculate from (3.58)

$$\mathrm{Tr}\,(-1)^{2J_L^3 + 2J_0} y^{L_0 + J_L^3} z^{L_0 + J_R^3} w^{J_0} = \frac{1 - yz}{1 - y} \sum_{r,s=0}^{3} (-1)^{r+s} h^{r,s} y^{\delta_{r,s} + \frac{1}{2}} w^{\frac{r-s}{2}} \quad (3.64)$$

where

$$
\delta_{r,s} \equiv \left\{ \begin{array}{ll} 0, & r + s = \text{odd}, \\ \frac{1}{2}, & r + s = \text{even}. \end{array} \right.
\tag{3.65}
$$

There is no essential difficulty in generalizing to the case $B \neq 0$, but the formula for the D0-brane index would be messier. So we will stay with $B = 0$ in the discussions below.

We can define an index that includes all multiple D0-brane states,

$$
\text{Tr}(-1)^{2J_L^3 + 2J_0} y^{L_0 + J_L^3} z^{L_0 + J_R^3} w^{J_0} q^N
\tag{3.66}
$$

where N is the total D0-brane charge. For a bound state of k D0-branes, the only change in the index is that the magnetic field B is effectively replaced by kB.

The full index of multi-D0-branes in $AdS_2 \times S^2 \times CY_3$ is

$$
\begin{aligned}
&\text{Tr}(-1)^{2J_L^3 + 2J_0} y^{L_0 + J_L^3} z^{L_0 + J_R^3} w^{J_0} q^N \\
&= \prod_{k=1}^{\infty} \prod_{r,s=0}^{3} \prod_{n=0}^{\infty} \left(\frac{1 - q^k y^{n + \delta_{r,s} + \frac{3}{2}} z w^{\frac{r-s}{2}}}{1 - q^k y^{n + \delta_{r,s} + \frac{1}{2}} w^{\frac{r-s}{2}}} \right)^{(-1)^{r+s} h^{r,s}}
\end{aligned}
\tag{3.67}
$$

where we traced over all possible numbers of various D0-brane bound states. In deriving the above expression we used the fact that the fermion number $(-1)^F$ of the chiral primaries is essentially $(-1)^{r+s}$, and for the descendants $(-1)^F$ changes according to $(-1)^{2J_L^3}$.

If we set $w = 1$ and $z = 1$, the index (3.67) can be written

$$
\begin{aligned}
&\text{Tr}(-1)^{2J_L^3 + 2J_0} y^{L_0 + J_L^3} q^N \\
&= \prod_{k=1}^{\infty} \left(1 - q^k y \right)^{-h_{even}} \left(1 - q^k y^{\frac{1}{2}} \right)^{h_{odd}}
\end{aligned}
\tag{3.68}
$$

An interesting special case is $y = e^{2\pi i}$. (3.68) then becomes

$$
\text{Tr}(-)^{F + 2J_0} q^N = \prod_{k=1}^{\infty} \left(1 - q^k \right)^{-h_{even}} \left(1 + q^k \right)^{h_{odd}}.
\tag{3.69}
$$

In fact (3.69) is true for general magnetic flux B on the S^2, if we replace q by $\tilde{q} = (-1)^B q$.

3.7 Wrapping the horizon

In this section we construct the superconformal quantum mechanics for a non-abelian configuration of N D0-branes which via the Myers effect becomes a D2 brane with N units of worldvolume magnetic flux wrapping the horizon. These configurations play an important role in black hole entropy as described in [5].

The low energy limit of the world-volume theory of the D2-brane expanded near its geodesic can be described again by a $\mathcal{N} = 4$ superconformal quantum mechanics. Since it wraps the S^2, the D2-brane sees an $AdS_2 \times CY_3$ target space. There are no world-volume zero modes of the gauge field. Due to the background RR 4-form flux of the form

$F^{(4)} = \omega_{S^2} \wedge J$, where J is the Kähler form on CY_3, the S^2-wrapped D2-brane is effectively charged under a magnetic field $F_{CY} = 4\pi Q J$ on the CY (here Q is the graviphoton charge, not to be confused with supercharges). The supercharges are of the form

$$
Q_\alpha = (QT_{Q,N})^{-\frac{1}{2}} \left[\frac{1}{2}\lambda_\alpha \hat{P}_\xi - \frac{i}{\xi} L^\eta_{\alpha\beta}\lambda^\beta + \frac{i}{4\xi}\bar{\lambda}_\alpha \lambda^2 + \frac{i}{4\xi}\lambda_\alpha \right]
$$
$$
+ \left(\frac{Q}{T_{Q,N}} \right)^{\frac{1}{2}} \left[\frac{\sqrt{2}}{\xi}\eta^a_\alpha(\hat{P}^{CY}_a + A_a) - 4\pi Q^2 \frac{i}{\xi}\lambda_\alpha \right],
$$
$$
\bar{Q}_\alpha = (QT_{Q,N})^{-\frac{1}{2}} \left[\frac{1}{2}\bar{\lambda}_\alpha \hat{P}_\xi - \frac{i}{\xi} L^\eta_{\alpha\beta}\bar{\lambda}^\beta - \frac{i}{4\xi}\bar{\lambda}^2 \lambda_\alpha - \frac{i}{4\xi}\bar{\lambda}_\alpha \right]
$$
$$
+ \left(\frac{Q}{T_{Q,N}} \right)^{\frac{1}{2}} \left[\frac{\sqrt{2}}{\xi}\bar{\eta}^{\bar{a}}_\alpha(\hat{P}^{CY}_{\bar{a}} + A_{\bar{a}}) + 4\pi Q^2 \frac{i}{\xi}\bar{\lambda}_\alpha \right],
$$
$$
S_\alpha = (QT_{Q,N})^{\frac{1}{2}} 2\xi\lambda_\alpha, \quad \bar{S}_\alpha = (QT_{Q,N})^{\frac{1}{2}} 2\xi\bar{\lambda}_\alpha, \tag{3.70}
$$

where A is the gauge connection on the CY, with field strength $dA = F_{CY} = 4\pi Q J$. $T_{Q,N} = \sqrt{(4\pi Q^2)^2 + N^2}$ is the mass of the D2-brane with N D0-brane charges. Comparing with (3.45), we dropped the L^{S^2} terms, replaced the $\hat{P}^{CY}$ by the gauge covariant derivative, and added $-iK^{-1}S_\alpha = -i\xi^{-1}\lambda_\alpha$ to Q_α and $iK^{-1}\bar{S}_\alpha = i\xi^{-1}\bar{\lambda}_\alpha$ to $\bar{Q}_\alpha$. The last modification is crucial for $\{Q_\alpha, \bar{Q}_\beta\} = 2\epsilon_{\alpha\beta}H$ to hold for some Hamiltonian H. Most of the superalgebra is the same as $D(2,1;0)$ given by (3.46), except for

$$
\{Q_\alpha, \bar{S}_\beta\} = \epsilon_{\alpha\beta}(D - 8\pi i Q^3) - 2iT_{\alpha\beta},
$$
$$
\{S_\alpha, \bar{Q}_\beta\} = \epsilon_{\alpha\beta}(D + 8\pi i Q^3) + 2iT_{\alpha\beta}, \tag{3.71}
$$

where $T_{\alpha\beta} = L^\eta_{\alpha\beta} + L^\lambda_{\alpha\beta}$. Writing the supercharges as $SU(2)_R \times SU(2)_L$ doublets, $Q_{\alpha+} \equiv Q_\alpha, Q_{\alpha-} \equiv \bar{Q}_\alpha$, etc., we have

$$
\{Q_{\alpha A}, S_{\beta B}\} = \epsilon_{\alpha\beta}\epsilon_{AB}D - 2i\epsilon_{AB}T_{\alpha\beta} - i\epsilon_{\alpha\beta}R_{AB} \tag{3.72}
$$

where $R_{+-} = R_{-+} = 8\pi Q^3, R_{++} = R_{--} = 0$. The commutators of $G^{\alpha A}_{\pm\frac{1}{2}} \equiv \frac{1}{\sqrt{2}}(Q^{\alpha A} \mp iS^{\alpha A})$ are

$$
\{G^{\alpha A}_{\pm\frac{1}{2}}, G^{\beta B}_{\pm\frac{1}{2}}\} = \epsilon^{\alpha\beta}\epsilon^{AB}(H - K \mp iD),
$$
$$
\{G^{\alpha A}_{\frac{1}{2}}, G^{\beta B}_{-\frac{1}{2}}\} = \epsilon^{\alpha\beta}\epsilon^{AB}(H + K) + 2\epsilon^{AB}T^{\alpha\beta} + \epsilon^{\alpha\beta}R^{AB}. \tag{3.73}
$$

This is $SU(1,1|2)$ algebra with a central extension R_{AB}, which explicitly breaks $SU(2)_L$ R-symmetry.

An interesting problem is to compute an index that counts the chiral primaries of this superconformal quantum mechanics, analogous to the one in section 3.6. It is shown in a companion paper [5] that these chiral primaries have the right degeneracies to account for the leading entropy of D4-D0 black holes, suggesting that the horizon wrapped D2-branes may describe the miscrostates of the black hole.

Acknowledgments

A. Representations of $\mathcal{N} = 4$ superconformal algebras

In this appendix we describe some properties of representations of the superconformal algebras related to D0-branes in $AdS_2 \times S^2$.

A.1 $D(2,1;\alpha)$ with $\alpha \neq 0$

The superconformal algebra $D(2,1;\alpha)$ takes the form

$$
\{G^{\alpha A}_{\pm\frac{1}{2}}, G^{\beta B}_{\pm\frac{1}{2}}\} = \epsilon^{\alpha\beta}\epsilon^{AB}L_{\pm 1},
$$
$$
\{G^{\alpha A}_{\frac{1}{2}}, G^{\beta B}_{-\frac{1}{2}}\} = \epsilon^{\alpha\beta}\epsilon^{AB}L_0 + \gamma\epsilon^{\alpha\beta}T^{AB}_L + (1-\gamma)\epsilon^{AB}T^{\alpha\beta}_R. \tag{A.1}
$$

where $\gamma = \alpha/(1+\alpha)$. Note that $(G^{\alpha A}_{\pm\frac{1}{2}})^\dagger = \epsilon_{\alpha\beta}\epsilon_{AB}G^{\beta B}_{\mp\frac{1}{2}}$. A highest weight state $|h\rangle$ is annihilated by all the $G^{\alpha A}_{\frac{1}{2}}$'s. $|h\rangle$ is a chiral primary if it is further annihilated by $G^{++}_{-\frac{1}{2}}$, or equivalently if $L_0|h\rangle = (\gamma j_L + (1-\gamma)j_R)|h\rangle$. Chiral primaries are contained in short representations, with $SU(2)_L \times SU(2)_R$ spin content [23]

$$
(j_L, j_R)_S \rightarrow (j_L, j_R)_0 + \left(j_L + \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{1}{2}} + \left(j_L - \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{1}{2}} + \left(j_L - \frac{1}{2}, j_R + \frac{1}{2}\right)_{\frac{1}{2}}
$$
$$
+ (j_L - 1, j_R)_1 + (j_L, j_R - 1)_1 + (j_L, j_R)_1 + \left(j_L - \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{3}{2}} \tag{A.2}
$$

where the subscripts denote the increase of L_0 value compared to the chiral primary $|h\rangle$, which is the highest weight state in $(j_L, j_R)_0$. The eight highest $SU(2) \times SU(2)$ states in (A.2) are obtained from $|h\rangle$ by acting with the three broken $G_{-1/2}$'s. In writing (A.2) we omitted $SL(2,\mathbf{R})$ descendants of $|h\rangle$ and the states obtained by acting on them with $G_{-\frac{1}{2}}$'s. These descendants have the spin content similar to a long representation. A general long representation $(j_L, j_R)_{long}$ satisfy the unitarity bound $L_0 \geq \gamma j_L + (1-\gamma)j_R$, and has the spin content of short representations $(j_L, j_R)_S + (j_L + \frac{1}{2}, j_R + \frac{1}{2})_S$.

A general index that vanishes for long representations and keeps informations about short representations of $D(2,1;\alpha)$ is of the form

$$
\text{Tr}(-1)^{2J^3_L} y^{L_0 \pm J^3_L} z^{L_0 \pm J^3_R} \tag{A.3}
$$

This will be an index for the representations of superconformal algebras discussed below as well.

A.2 $D(2,1;0)$

A highest weight state $|h\rangle$ of $D(2,1;0)$ that is annihilated by $G^{++}_{-\frac{1}{2}}$ is necessarily annihilated by $G^{+-}_{-\frac{1}{2}}$. These are the chiral primaries of $D(2,1;0)$, which generate short representations with spin content

$$(j_L, j_R)_S \to (j_L, j_R)_0 + \left(j_L - \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{1}{2}} + \left(j_L + \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{1}{2}} + (j_L, j_R - 1)_1 \quad \text{(A.4)}$$

A long representation has spin content of four short representations

$$(j_L, j_R)_{long} \to (j_L, j_R)_S + \left(j_L - \frac{1}{2}, j_R + \frac{1}{2}\right)_S + \left(j_L + \frac{1}{2}, j_R + \frac{1}{2}\right)_S + (j_L, j_R + 1)_S. \quad \text{(A.5)}$$

A.3 $SU(1,1|2)$

$D(2,1;0)$ is $SU(1,1|2)$ together with its outer automorphism $SU(2)_L$. The R-symmetry of $SU(1,1|2)$ is denoted $SU(2)_R$. Similar to $D(2,1;0)$, a short representation of $SU(1,1|2)$ has $U(1)_L \times SU(2)_R$ spin content

$$(j_L^3, j_R)_S \to (j_L^3, j_R)_0 + \left(j_L^3 - \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{1}{2}} + \left(j_L^3 + \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{1}{2}} + (j_L^3, j_R - 1)_1 \quad \text{(A.6)}$$

where we labelled the J_L^3 charge of $SU(2)_L$ for each $SU(2)_R$ multiplet. A long representation again consists of four short representations.

A.4 $SU(1,1|2)$ **with central extension**

As discussed in section 3.7, the D2-brane wrapped on the S^2 has a superconformal algebra which is a central extension of $SU(1,1|2)$. We shall denote it by $SU(1,1|2)_Z$. The anticommutators are given by

$$\left\{G^{\alpha A}_{\pm\frac{1}{2}}, G^{\beta B}_{\pm\frac{1}{2}}\right\} = \epsilon^{\alpha\beta}\epsilon^{AB}L_{\pm 1},$$
$$\left\{G^{\alpha A}_{\frac{1}{2}}, G^{\beta B}_{-\frac{1}{2}}\right\} = \epsilon^{\alpha\beta}\epsilon^{AB}L_0 + \epsilon^{AB}T^{\alpha\beta} + \epsilon^{\alpha\beta}R^{AB}. \quad \text{(A.7)}$$

where R^{AB} is a constant symmetric tensor, with $R^{++} = R^{--} = 0$, $R^{+-} = R^{-+} = r > 0$. Now a chiral primary state $|h\rangle$ can only be defined to be annihilated by $G^{++}_{-\frac{1}{2}}$ but not by $G^{+-}_{-\frac{1}{2}}$. It saturates the unitarity bound $L_0|h\rangle = (j_R + r)|h\rangle$. The short representations have spin content similar to (A.2), except that we can only label them by $U(1)_L \times SU(2)_R$ multiplets,

$$(j_L^3, j_R)_S \to (j_L^3, j_R)_0 + \left(j_L^3 + \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{1}{2}} + \left(j_L^3 - \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{1}{2}} + \left(j_L^3 - \frac{1}{2}, j_R + \frac{1}{2}\right)_{\frac{1}{2}}$$
$$+ (j_L^3 - 1, j_R)_1 + (j_L^3, j_R - 1)_1 + (j_L^3, j_R)_1 + \left(j_L^3 - \frac{1}{2}, j_R - \frac{1}{2}\right)_{\frac{3}{2}}. \quad \text{(A.8)}$$

Note that the highest weight state in $(j_L^3 - \frac{1}{2}, j_R + \frac{1}{2})_{\frac{1}{2}}$ also saturates the unitarity bound, but it is not annihilated by $G_{\frac{1}{2}}^{-+}$. A long representation $(j_L^3, j_R)_{long}$ of $SU(1,1|2)_Z$ has the spin content of two short representations $(j_L^3, j_R)_S + (j_L^3 + \frac{1}{2}, j_R + \frac{1}{2})_S$.

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